

Axion model in extra dimensions with TeV scale gravity

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Abstract

A simple axion model is proposed in the scenario of large extra dimensions where gravity scale is as low as 1 TeV. To obtain an intermediate-scale decay constant of the axion, the axion is assumed to live in a sub-spacetime (brane) of the whole bulk. In this model there appear Kaluza-Klein modes of the axion which have stronger interaction than those of the graviton. The axion brane plays a role of absorber of the graviton Kaluza-Klein modes. Because of these reasons phenomenology and cosmology of the axion become highly non-trivial. We discuss various cosmological constraints as well as astrophysical ones and show that the model is viable for certain choices of the dimensionality of the axion brane. The structure of the model proposed here provides a viable realization of the fat brane idea to relax otherwise very severe cosmological constraints.

I. INTRODUCTION

It has been suggested that the fundamental scale of nature can be as low as TeV, whereas the largeness of the effective Planck scale or the weakness of the gravity in a long distance can be explained by introducing large extra dimensions [1–4]. When there exist n of such extra dimensions, the relation between the gravitational constant $8\pi G_N = 1/M_{pl(4)}^2$ in 4 dimensions and the fundamental scale $M_{pl(4+n)}$ in $(4+n)$ dimensions is given

$$M_{pl(4)}^2 \sim V_n M_{pl(4+n)}^{n+2}, \quad (1)$$

where V_n is the volume of the extra dimensional space. For $M_{pl(4+n)} \sim 1$ TeV, the size of the extra dimensions r_n is computed as

$$r_n \sim V_n^{1/n} \sim 10^{\frac{32}{n}-6} M_{\text{TeV}}^{-\frac{2}{n}-1} \text{MeV}^{-1} \sim 10^{\frac{32}{n}-17} M_{\text{TeV}}^{-\frac{2}{n}-1} \text{cm}, \quad (2)$$

where $M_{\text{TeV}} \equiv M_{pl(4+n)}/\text{TeV}$. The case $n = 1$ is excluded because the gravitational law would change at macroscopic level, but the cases $n \geq 2$ are allowed by gravity experiments.

There are also astrophysical and cosmological bounds [3] because there are a tower of graviton Kaluza-Klein (KK) modes which contribute to the supernova cooling [5], the total mass of the universe and the late-time photon production [6] *etc.* We may avoid some of these problems by assuming an extremely low reheating temperature of the early universe.

To have an inflation model with such a low temperature is already a tough subject, especially with the cosmological moduli problem [7]. Furthermore, we need baryon asymmetry after the inflation while the temperature is quite low and we knew that the process which generates baryon asymmetry should not induce proton decay. It is better to have higher reheating temperature for many reasons, but it is very hard to avoid known restrictions.

There is another problem in TeV scale gravity. The simplest model of this type would not provide intermediate scales which are necessary to explain phenomenological issues like, the apparent gauge coupling unification [8], small neutrino masses and the strong CP problem. It was pointed out [3] that particles dwelling in the extra dimensions, other than graviton, can

have similar effective interaction terms in four-dimensional physics. This type of interaction is suppressed as gravity and thus give extremely weak interaction between bulk matter and normal matters. For the neutrino, there have been many discussions and researches about how to extend the minimal setting to obtain the small neutrino masses and mixing [9–11], but the axion as a solution of the strong CP problem has not been studied thoroughly in this context. In this paper we will propose an axion model in TeV scale gravity for various numbers of the extra dimensions, with special emphasis on cosmological constraints: the addition of the axion in the large-extra-dimension model substantially alters the cosmology.

In the following sections, we will use the convention $M_{pl} \equiv M_{pl(4)}$ = reduced Planck mass and $M_* \equiv M_{pl(4+n)}$ = fundamental scale.

II. PQ SCALE IN EXTRA-DIMENSION

If an axion is a boundary field confined on 4 dimensions, the Peccei-Quinn (PQ) scale f_{PQ} is bounded by $M_* \sim 1$ TeV. To obtain a higher PQ scale, the axion field has to be inevitably a bulk field. If it lives in the whole $(4+n)$ dimensional bulk, the PQ scale will be $f_{PQ} \sim M_{pl}$.

However, the damped coherent oscillation of the axion vacuum at the early universe with $f_{PQ} \sim M_{pl}$ would overclose the universe. A conventional argument gives an upper bound $f_{PQ} \leq 10^{12}$ GeV. Even when an entropy production takes place after the QCD phase transition (*e.g.* the reheating temperature is smaller than ~ 1 GeV), f_{PQ} cannot be much larger than 10^{15} GeV.

In Ref. [12], it was shown that the late thermal inflation can raise bound of the PQ scale up to 10^{15} GeV. This argument can be applied to our case, provided that the coherent oscillation of the inflaton is followed by the reheating process. Then the bound on f_{PQ} is

$$f_{PQ} < 10^{15} \text{GeV} \left(\frac{h}{0.7} \right) \left(\frac{\pi/2}{\theta} \right) \left(\frac{\text{MeV}}{T_R} \right)^{1/2}, \quad (3)$$

where h is from Hubble constant in units of $100 \text{km sec}^{-1} \text{Mpc}^{-1}$, θ is the initial value of PQ

vacuum angle and T_R is the reheating temperature after inflation. Thus the axion model suffers from the over-closure problem if there is no other scale than M_* and M_{pl} .

In this paper, we propose that a natural way to realize an intermediate scale axion is to make it live in a $(4 + m)$ dimensional sub-spacetime $((3 + m)$ brane) ($m < n$) of the whole $(4 + n)$ dimensional bulk. The idea of using sub-spacetime to realize an intermediate scale already appeared in Ref. [10] in the context of neutrinos.

Let χ be a complex scalar field which contains PQ axion in $4+m$ dimension; \tilde{a} . If the axion field lives only on $4 + m$ dimensional sub-spacetime where $m < n$ and the volume of extra-dimension is V_m ,

$$\mathcal{L}_\chi = \int dx^{4+m} \partial^M \chi^* \partial_M \chi + \int dx^4 \frac{\tilde{a}(x^A = 0)}{\langle \chi \rangle} F \tilde{F} \quad (4)$$

where x^A means an extra-dimension coordinate. Assuming that the vacuum expectation value of the χ field does not depend on the extra-dimension coordinates, we obtain

$$\begin{aligned} f_{PQ} &\sim \sqrt{V_m} \langle \chi \rangle \sim r_n^{m/2} M_*^{1+m/2} \sim M_* \left(\frac{M_{pl}}{M_*} \right)^{m/n} \\ &\sim 10^{3(1+5m/n)} M_{\text{TeV}}^{1-m/n} \text{GeV}. \end{aligned} \quad (5)$$

Here we have defined the 4-D axion field as $a = \sqrt{V_m} \tilde{a}(x^A = 0)$ and assumed that the size r_n is common for all extra dimensions.

A lower bound of f_{PQ} comes from astrophysical observations, e.g. red giant and supernova cooling by axion emission. It is known that f_{PQ} should be larger than 10^9GeV . In extra dimension physics, KK modes also contribute to supernova cooling if their masses are smaller than the core temperature ($\sim 30 \text{MeV}$). We discuss it in the next section.

To have $10^9 \text{ GeV} < f_{PQ} \leq 10^{15} \text{ GeV}$, we need $2/5 < m/n \leq 4/5$. Possible sets of (m, n) with f_{PQ} where $M_* = 1 \text{ TeV}$ and 10 TeV can be found in Table I, II.

III. LABORATORY AND ASTROPHYSICAL CONSTRAINTS

Experiments on detecting axion from the nuclear reactor and the sun give bounds on PQ scale $< 10^4 \text{ GeV}$ (Lab), $10^4 \text{ GeV} < f_{PQ} < 10^6 \text{ GeV}$ from the sun. However this limit

strongly depends on the photon axion coupling and method of detecting axion. Furthermore the laboratory bound is much weaker than the astrophysical bound.

The strongest bound from astrophysics is the Supernova cooling. In SN1987A observation, it was calculated that

$$\frac{1}{f_{PQ}^2} < 10^{-18} \text{ GeV}^{-2}. \quad (6)$$

Since the axion KK modes interact exactly the same way as the conventional axion, the effective interaction of the KK modes at the core temperature $T \simeq 30 \text{ MeV}$ is

$$\frac{1}{f_{PQ}^2} \times (Tr_n)^m \sim \frac{T^m}{M_*^{m+2}} < 10^{-18} \text{ GeV}^{-2}. \quad (7)$$

This gives a bound on the fundamental scale

$$M_* > (10^{18} \times 0.03^m)^{\frac{1}{m+2}} \text{ GeV}. \quad (8)$$

For $m = 1$, $M_* > 300 \text{ TeV}$ ¹ and $m = 2$, $M_* > 5 \text{ TeV}$. $M_* \sim 1 \text{ TeV}$ is allowed only if $m > 2$. This suggests that the number of extra-dimension should be at least 3 to include PQ mechanism as a solution of strong CP problem in TeV scale gravity model.

In near future, high energy accelerator experiments may probe the axion KK mode emission. The graviton KK mode signals in collider were discussed in detail recently. [14] The scattering cross section of the graviton KK mode emission from high energy scattering with center of mass frame energy \sqrt{s} is

$$\sigma \propto \frac{1}{M_{pl}^2} (\sqrt{s} r_n)^n \sim \left(\frac{\sqrt{s}}{M_*} \right)^{n+2} \frac{1}{s}, \quad (9)$$

while the KK axion production is

$$\sigma \propto \frac{1}{f_{PQ}^2} (\sqrt{s} r_n)^m \sim \left(\frac{\sqrt{s}}{M_*} \right)^{m+2} \frac{1}{s}. \quad (10)$$

¹It was argued that there arise large quantum corrections in the case of one extra dimension in general, which would destabilize the gauge hierarchy [13]. This argument would exclude the $m = 1$ case.

Since the energy dependence of the axion KK mode cross-section is different from the graviton KK mode cross-section, it might be possible to detect this difference at TeV scale collider experiments.

IV. THERMAL PRODUCTION OF AXION KK MODE

Since the axion dwells in the extra-dimensional brane, their masses are proportional to r_n^{-1} and they have stronger couplings than the graviton KK modes to the normal matters in general cases.

If there is no hidden particle which couples to the axion or the mass of the KK mode axion is lower than the sum of three pion masses ~ 500 MeV, the main decay channel of light KK axion is to two photons. The life-time of each KK mode for a given (n, m) can be found in Table I, II.

$$\Gamma_{a_{KK} \rightarrow 2\gamma} \simeq \frac{C_{a\gamma}^2}{64\pi} \left(\frac{\alpha}{\pi}\right)^2 \frac{m_A^3}{f_{PQ}^2} \sim 3 \cdot 10^{-8} C_{a\gamma}^2 \frac{m_A^3}{f_{PQ}^2}, \quad (11)$$

where m_A is the mass of the axion KK mode and $C_{a\gamma}$ is the model dependent axion-photon coupling which is usually within 0.1 to 1. This decay can be cosmologically dangerous. For instance, for $f_{PQ} = 10^{12}$ GeV and $m_A = 1$ MeV, life time of KK mode is $\tau_A \sim 10^{17}$ sec, which is about the age of the universe.

The graviton KK modes have similar cosmological problems because they can overclose our universe or decay into the photons at a late stage of cosmological evolution. Originally it was suggested that a “fat brane” [3] can solve the cosmological problems by absorbing most of the decay products of the KK modes. However massless particles in the higher dimensional brane *are not* massless in our four-dimensional universe since they have momenta in the extra dimensions which appear masses in our universe [5]. Another way to avoid this difficulty is assuming a large number of four-dimensional lattices in the bulk. But we need at least 10^6 *empty* universes. Or we should assume the existence of a four-dimensional hidden sector which has 10^6 times more degrees of freedom than those of the standard model while they should not be produced significantly by the reheating process after the inflation.

If we add an axion as a “brane particle” in this model, the graviton “bulk particle” will decay to the “brane” axion more efficiently, since its decay width will be enhanced by factor $(M_G r_n)^m$. The graviton KK mode with mass m_G will have decay width to the axion

$$\Gamma(g_{KK} \rightarrow 2a) \sim \mathcal{O}(10^{-3}) \frac{m_G^3}{M_{pl}^2} \times (m_G r_n)^m. \quad (12)$$

After some period, instead of the massive graviton KK fields, we will have the axion KK fields with about same masses. Since the massive axion KK mode can decay into the photon pairs and the life time of the graviton KK mode becomes much shorter, the primordial graviton KK mode will not over-close the universe. Instead, it will contribute to the cosmological background radiation. This can be a severe constraint to the axion model. Also axions can be produced thermally during the reheating process. For these reasons, we have to check whether the axion model can survive the cosmological constraints.

To estimate the constraints for various cases, we calculate the amount of the thermal axion produced at the reheating temperature T_R and the axion KK modes from the graviton KK modes decay. In Appendix A, we derived the Boltzmann Eq. for the yield $Y = \rho/s$ for four different sources of the axion KK modes:

I. decay of graviton KK modes from the inverse decay, $(2\gamma, e^+e^-, \bar{\nu}\nu \rightarrow g_{KK})$; the energy of the KK modes are concentrated on $m_A \sim T_R$,

II. decay of graviton KK modes produced from the scattering, $(e\gamma \rightarrow eg_{KK}, e^+e^- \rightarrow \gamma g_{KK})$; this contribution is significant only if $m_A < T_R$,

III. the axion KK mode from the pion scattering $(\pi\pi \rightarrow \pi a_{KK})$; this process dominates if $T_R > 10 \text{ MeV}$

IV. the axion KK mode from the two photon inverse decay $(2\gamma \rightarrow a_{KK})$; this gives significant contribution when $m_A \sim T_R$.

For each case,

$$Y_1 \simeq 3 \cdot 10^{-23} \left(\frac{T_R}{100 \text{ MeV}} \right) A_1, \quad (13)$$

$$Y_2 \simeq 2 \cdot 10^{-23} \left(\frac{T_R}{100 \text{ MeV}} \right) A_2, \quad (14)$$

$$Y_3 \simeq 6 \cdot 10^{-10} \left(\frac{10^{12} \text{GeV}}{f_{PQ}} \right)^2 \left(\frac{T_R}{100 \text{MeV}} \right)^3 A_3, \quad (15)$$

$$Y_4 \simeq 2 \cdot 10^{-16} \left(\frac{10^{12} \text{GeV}}{f_{PQ}} \right)^2 \left(\frac{T_R}{100 \text{MeV}} \right) A_4 \quad (16)$$

where

$$A_1 = \left(\frac{10}{g_*(T_R)} \right)^{3/2} \left(\frac{m_A}{T_R} \right)^3 \quad (17)$$

$$A_2 = \left(\frac{10}{g_*(T_R)} \right)^{3/2} \left(\ln \left(\frac{T_R^3}{m_A^2 m_e} \right) - 0.8 \right) \quad (18)$$

$$A_3 = C_{a\pi}^2 \left(\frac{10}{g_*(T_R)} \right)^{3/2} \left(\frac{I(T_R)}{1000} \right) \quad (19)$$

$$A_4 = C_{a\gamma}^2 \left(\frac{10}{g_*(T_R)} \right)^{3/2} \left(\frac{m_A}{T_R} \right)^3. \quad (20)$$

For the details of these calculations and the definition of function $I(T)$, see appendix A (we present the numerical plot of $I(T)$ in Fig. 5).

V. COSMOLOGICAL CONSTRAINTS

In this section we would like to discuss various cosmological constraints on the model for a given (m, n) set. Notice that in Table I and II, $n \geq 5$ in both $M_* = 1$ and 10 TeV case and $n = 4$ in 1 TeV are cosmologically safe if T_R is low enough (~ 1 MeV). If the minimal KK mode mass is greater than 1 MeV, the KK modes are not generated in the thermal bath of such a low reheating temperature. On the other hand $m \leq 2$ in $M_* = 1$ TeV and $m = 1$ in $M_* = 10$ TeV is forbidden by the astrophysical bound. The cases $n = 4, m = 3$ at $M_* = 1$ TeV and $n = 3, m = 2$ at $M_* = 10$ TeV are not trivially allowed or ruled out by the cosmological constraints. Details on these cases are discussed in Appendix B.

A. Big bang nucleosynthesis

At the temperature of the universe around 1 MeV, it is required that there should not be additional particles which contribute to the energy density significantly. Otherwise ^4He

would be produced more than what is observed now because the universe should expand faster than the standard scenario.

We apply a rather loose bound that the energy density contribution by the KK mode should be smaller than one neutrino energy density at $T = 1$ MeV. At high $T_R (> 10$ MeV), axion KK mode production dominated by process III. In case III, one may practically have maximal mass

$$m_1 \equiv \max\{m_\pi, T_R\} \quad (21)$$

for the KK mode which is produced in the thermal bath.

Thus the bound from BBN is

$$\begin{aligned} \left. \frac{\rho_A}{s} \right|_{BBN} &\simeq D \int_{m_0}^{m_1} dm_A \times (m_A r_n)^m Y_A \\ &\simeq m_1 Y_3(m_1 r_n)^m \simeq 2 \cdot 10^{-3} \frac{M_{pl} T_R^3 m_1^{m+1}}{M_*^{m+2} f_\pi^2} A_3 < 0.1 \text{ MeV}, \end{aligned} \quad (22)$$

where D is normalization constant which is equal to m in torus compactification with universal distance r_n . In our calculations we restrict ourselves to this case.

If $T_R \sim m_\pi$, approximately

$$M_* > 10^{\frac{20}{m+2}} \times m_\pi. \quad (23)$$

For $m = 1$, this reads $M_* > 600$ TeV, and for $m = 2$, $M_* > 15$ TeV. But if the reheating temperature is as low as 10 MeV, this bound is not important since $A_3(T_R)$ is suppressed exponentially.

B. Over-closure of Universe

The total energy of the axion KK modes at present must not exceed the critical density:

$$\rho_A < \rho_c = 3 \cdot 10^{-6} s_0 h^2 \text{ MeV}, \quad (24)$$

where $s_0 \simeq 3000 \text{ cm}^{-3}$ is the entropy of the present universe.

For the case that the KK modes decay into some relativistic particles, we can divide the bound in two parts; decay before the present time and do not decay till now:

$$\frac{\rho_A}{s_0} \simeq D \int_{m_0}^{m_2} dm_A (m_A r_n)^m Y_A + D \int_{m_2}^{m_1} dm_A (m_A r_n)^m \frac{Y_A T_0}{T(m_A)} < 3 \cdot 10^{-6} h^2 \text{ MeV}, \quad (25)$$

where the axion KK mode with mass m_2 decays at the present time, and m_1 is defined in Eq. (21).

C. Light element destruction

Energetic photons from heavy axion KK modes which decay after 10^4 sec can destroy the light elements made during the nucleosynthesis. Therefore there are several bounds on the density of the KK modes which weigh than 10 MeV. They are [15]

$$\frac{\rho_A}{s} \leq 10^{-12} \text{ GeV} \quad (26)$$

for $\tau_A \geq 10^7$ sec,

$$\frac{\rho_A}{s} \leq 10^{-6} - 10^{-10} \text{ GeV} \quad (27)$$

for $10^4 < \tau_A \leq 10^7$ sec. Usually these bounds are less important than cosmological microwave background bound given below.

D. Cosmological Microwave Background Radiation

If the massive KK modes decay after $\tau_A \geq 10^6$ sec but before the recombination era, the produced photons may give a distortion of the cosmological microwave background radiation. The COBE observation gives a bound [16]

$$\frac{\Delta \rho_\gamma}{s} \leq 2.5 \times 10^{-5} T_D \quad (28)$$

where T_D is temperature at KK mode decay.

E. Diffuse photon background

Observations of diffuse photon backgrounds at the present universe give upper bounds on additional contributions to photon spectrum. For example, for the energy range $800 \text{ keV} < E < 30 \text{ MeV}$ [17]

$$\frac{d\mathcal{F}}{d\Omega} \simeq E \times A (E/E_0)^{-\alpha} \simeq 78 \left(\frac{E}{1\text{keV}} \right)^{-1.4}. \quad (29)$$

Constraints on other ranges of the photon energy can be found, e.g. in Ref. [18]

Theoretical prediction is

$$\frac{d\mathcal{F}}{d\Omega} = \frac{n_A c}{4\pi} \times Br \quad (30)$$

for the life-time of KK mode shorter than the age of the universe, and

$$\frac{d\mathcal{F}}{d\Omega} \sim Br \times \frac{n_A c}{4\pi} \frac{\Gamma_{a_{KK} \rightarrow 2\gamma}}{Br H_0} \left(\frac{2E}{m_A c^2} \right)^{3/2} (m_A r_n)^m \quad (31)$$

for its life time longer than the age of the universe [19]. Here we have introduced Br as a branching ratio of axion decay into two photons.

In our brane picture, there is *a priori* no reason that the axion brane contains only our four-dimensional wall. Rather it will be natural to imagine that there is a parallel universe(s) or another four-dimensional wall in the brane. Or one can just imagine that there are some unknown particles on our wall itself. Then one can consider the situation that thermally produced KK modes of graviton will mainly decay into the axion in the brane and axions both from the graviton decay and the thermal production will decay into the parallel wall if this wall has some kind of QCD and/or $U(1)$ type of interactions. (Or it can decays into some hidden QCD/ $U(1)$ fields in our universe.) Since the axion decay width is highly suppressed by $\left(\frac{\alpha}{\pi}\right)^2$ with α being the fine structure constant, it is easy to get a low branching ratio to decay into the photon, in other words, most of the axions decay into the other wall(invisible section) if the coupling constant, the color factor of the other gauge interaction, and/or the number of the fermions with PQ charges in the decay loop diagram in the other wall(invisible section) are large enough.

F. Results

Here we summarize the results we obtained. The reader should be referred to Appendix B for more detail. (We approximate $h = 0.7$)

I. $n = 4$, $m = 3$ and $M_* = 1$ TeV case:

1. BBN bound

$$T_R < 90 A_3^{-1/3} \text{ MeV} \simeq 80 \text{ MeV}. \quad (32)$$

2. Over-closure bound,

$$T_R < 12 \left(\frac{C_{a\gamma}^2}{A_3^2 Br} \right)^{\frac{1}{6}} \text{ MeV} \simeq 30 \text{ MeV} \text{ (for } Br = 1). \quad (33)$$

3. CMBR bound, (for $m_A \geq 100$ MeV and $T_R > 10$ MeV)

$$T_R < 2 \cdot 10^{-2} \left(\frac{C_{a\gamma}^2}{A_3^2 Br^3} \right)^{\frac{1}{6}} \text{ MeV}. \quad (34)$$

4. Diffused photon bound,

for $T_R > 10$ MeV,

$$T_R < 2 \cdot 10^{-2} \left(\frac{Br}{C_{a\gamma}} \right)^{-0.63} \left(\frac{m_A}{10 \text{ MeV}} \right)^{-0.53} A_3^{-1/3} \text{ MeV}, \quad (35)$$

for $T_R < 10$ MeV,

$$T_R < 0.3 Br^{-0.73} \text{ MeV}, \quad (36)$$

where $Br < \Gamma_{a \rightarrow 2\gamma}/H_0$, and

$$T_R < 5 C_{a\gamma}^{-0.48} \text{ MeV} \quad (37)$$

where $Br > \Gamma_{a \rightarrow 2\gamma}/H_0$.

II. $n = 3$, $m = 2$ and $M_* = 10$ TeV case:

1. BBN bound

$$T_R < 100 A_3^{-1/3} \text{ MeV} \simeq 90 \text{ MeV}. \quad (38)$$

2. Over-closure bound,

$$T_R < 28 \left(\frac{C_{a\gamma}^2}{A_3^2 Br} \right)^{\frac{1}{6}} \text{ MeV} \simeq 40 \text{ MeV (for } Br = 1). \quad (39)$$

3. CMBR bound,

$$T_R < 4 \cdot 10^{-2} \left(\frac{C_{a\gamma}^2}{A_3^2 Br^3} \right)^{\frac{1}{6}} \text{ MeV}. \quad (40)$$

4. Diffused photon bound,

for $T_R > 10 \text{ MeV}$,

$$T_R < 3 \cdot 10^{-2} \left(\frac{Br}{C_{a\gamma}} \right)^{-0.63} \left(\frac{m_A}{10 \text{ MeV}} \right)^{-0.2} A_3^{-1/3} \text{ MeV}, \quad (41)$$

for $T_R < 10 \text{ MeV}$,

$$T_R < 0.1 Br^{-1.2} \text{ MeV}, \quad (42)$$

where $Br < \Gamma_{a \rightarrow 2\gamma}/H_0$, and

$$T_R < 3 \text{ MeV} \quad (43)$$

where $Br > \Gamma_{a \rightarrow 2\gamma}/H_0$.

In the both cases, $n = 4$, $m = 3$ and $M_* = 1 \text{ TeV}$ and $n = 3$, $m = 2$ and $M_* = 10 \text{ TeV}$, life times of axion KK modes up to $m_A \sim 1 \text{ GeV}$ are quite long and so the constraint from the light element destruction will not give any further bound.

We also performed computer calculations on both $n = 4$, $m = 3$ and $M_* = 1 \text{ TeV}$ and $n = 3$, $m = 2$ and $M_* = 10 \text{ TeV}$ cases. Fig. 1 and 2 show that the DPBR is the stringent bound if the branching ratio is small. But in an extremely small branching ratio case, CMBR is dominant. This is because the life time of the axion KK modes becomes shorter than 10^{13} sec, and so the produced photons will disturb the CMBR spectrum. One can see that BBN is independent of the branching ratio. Note that these behaviors are consistent with what we observed from the analytical computations given above.

In Fig. 3 and 4, we present combined cosmological limits for all possible combinations of n, m in Table I and II. We find that the reheating temperature is allowed to be significantly large if the branching ratio to the photon is small enough.

VI. CONCLUSIONS

In this paper, we discussed the axion model in the extra dimensions whose PQ scale lies in an intermediate scale $f_{PQ} \leq 10^{15}$ GeV. This intermediate scale can be obtained by introducing a $3 + m$ dimensional brane in the $4 + n$ dimension bulk.

If we include the axion as a brane particle, it will change the phenomenology of the extra dimension physics, especially, cosmology. Since the graviton KK mode will decay into the axion KK mode, the over-closure problem is not as serious as the original model of Arkani-Hamed et.al. On the other hand, the argument from stars and supernova cooling will give a more strict bound on the axion production. Among other things, the most severe cosmological bound comes from photon emission through the decays of the KK modes of the axion. We found that the astrophysical argument restrict the number of the dimensionality of the sub-spacetime where the axion lives: $m > 2$ for $M_* = 1$ TeV and $m > 1$ for $M_* = 10$ TeV. The latter cosmological argument requires quite a low reheating temperature after the inflation.

To lift this bound, we can introduce the hidden matter/gauge fields to another four-dimensional wall (or even to our wall itself) which has much stronger coupling to axion and/or much more generations of particles, (or maybe much lower QCD phase transition scale, etc). This can significantly lower the branch ratio of the axion KK mode decay into photons.

The whole picture can be used to improve the original *fat brane* idea. This higher dimensional object plays a role of an absorber of the KK graviton modes. If the fat brane couples to four dimensional wall(s) with interaction stronger than gravity, the produced particles in the fat brane may then decay into relativistic particles on the four dimensional wall(s). This mechanism can solve the problem of the overclosure of the universe by the KK modes. Note that the fat-brane particles are not necessarily the axions as we discussed, but can be any other weakly interacting particles living in a higher dimensional brane. Moreover if the particles produced by the fat-brane particle decays do not contain photons or any

other cosmologically dangerous particles, then we can avoid other cosmological problems such as the ones related to the cosmic photon backgrounds. It is worth mentioning that some mechanisms of generating small neutrino masses have a similar structure, which will be discussed elsewhere [20]

In this paper, we did not consider an alternative solution to the strong CP problem such as a spontaneous CP breaking model. This typically requires very heavy quarks. If we want to keep all particles carrying standard-model gauge charges in four dimensions, the maximum scale we can have is the fundamental scale i.e. around TeV. For this reason, this type of model is not favorable in the extra-dimension scheme.

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APPENDIX A: ESTIMATION OF THE PRIMORDIAL AXION KK MODE DENSITY

The production rate of the axion KK mode from initial particle i and j can be calculated from Boltzmann Eq.,

$$\dot{n}_A + 3H(t)n_A = \sum_{ij} \langle \sigma v \rangle_{ij} n_i n_j, \quad (\text{A1})$$

where $n_A, H(t), \sigma, n_i$ represent axion KK mode number density, Hubble constant, scattering cross section and number density of initial particle i .

Using the relation $t = 0.5H(T)^{-1}$, we can convert the time parameter to inverse temperature $x \equiv m/T$, where $H(T) \simeq \left(\frac{g_*}{10}\right)^{1/2} \frac{T^2}{M_{pl}}$. If we assume that the KK modes are produced

by the particles in equilibrium, we can rewrite (A1) with the yield $Y \equiv n_A/s$. (s is entropy density $s = \frac{2\pi^2}{45} g_{*s} T^3$, where $g_{*s} \simeq g_*$ is approximately 10 for $1 \text{ MeV} < T < 100 \text{ MeV}$.)

$$\frac{dY}{dx} = \frac{x}{H(m)} \Gamma_A Y_{EQ} \quad (\text{A2})$$

where

$$\Gamma_A = n_{EQ} \sum_{ij} \langle \sigma v \rangle_{ij}. \quad (\text{A3})$$

A similar equation can be derived for the inverse decay case.

$$\frac{dY}{dx} = \frac{x}{H(m)} \Gamma(a_{KK} \rightarrow \text{ALL}) \left\langle \frac{m_A}{E_A} \right\rangle Y_{EQ} \quad (\text{A4})$$

The yield of the KK mode at the equilibrium Y_{EQ} is about $0.28/g_{*s}$ when initial particles are relativistic (or proportional to $\exp(-x)$ if they are non-relativistic). After integrating Eq.(A2) and (A4) from the reheating temperature T_R to present temperature, we will get result with form

$$Y \simeq \frac{\Gamma}{H(T_R)} Y_{EQ}(T_R) \sim 3 \cdot 10^{-2} \left(\frac{10}{g_*(T_R)} \right)^{3/2} \frac{M_{pl} \Gamma}{T_R^2}, \quad (\text{A5})$$

which can be used most of calculations reliably.

Let's estimate the sources of the axion KK mode production. The KK modes of axion can be produced from either the thermal graviton KK mode decay or initial thermal bath. We will classify four relevant cases.

Class I: The KK mode of graviton which has mass around the reheating temperature $m_G \simeq T_R$ generated dominantly through the inverse decay $\gamma\gamma \rightarrow g_{KK}$, $\bar{\nu}\nu \rightarrow g_{KK}$ and $e^+e^- \rightarrow g_{KK}$ [14]

$$\Gamma_{g_{KK} \rightarrow 2\gamma} = \frac{m_G^3}{80\pi M_{pl}^2} \quad (\text{A6})$$

$$\Gamma_{g_{KK} \rightarrow f\bar{f}} = \frac{m_G^3}{160\pi M_{pl}^2} \quad (\text{A7})$$

for initial spin averaged. For the tensor mode of graviton KK mode we should multiply 5 to Γ . This will generate

$$Y_1 \simeq 6 \times 10^{-4} \frac{T_R}{M_{pl}} A_1 \simeq 3 \cdot 10^{-23} \left(\frac{T_R}{100 \text{MeV}} \right) A_1. \quad (\text{A8})$$

where

$$A_1 = \left(\frac{10}{g_*(T_R)} \right)^{3/2} \left(\frac{m_A}{T_R} \right)^3. \quad (\text{A9})$$

Here we approximated that $m_A \simeq m_G$ after the graviton decay. This mode is most abundant at $m_A = T_R$ and decrease fast if $m_A \ll T_R$.

Class II: The KK modes which has much less mass than $m_G < T_R$ will be produced dominantly by the scattering processes $e^\pm \gamma \rightarrow g_{KK} e^\pm$, $e^+ e^- \rightarrow g_{KK} \gamma$. If we choose a limit that KK mode mass is less than T_R but greater than the electron mass, we can calculate the interaction rate from the amplitude presented in [14].

$$\Gamma_{e^\pm \gamma \rightarrow g_{KK} e^\pm} \simeq \langle \sigma v \rangle n_{EQ} \simeq \frac{\alpha}{M_{pl}^2} \left(\ln \frac{T_R^3}{m_G^2 m_e} - \frac{7}{8} \right) \times (0.3) T_R^3 \quad (\text{A10})$$

$$\Gamma_{e^+ e^- \rightarrow g_{KK} \gamma} \simeq \frac{\alpha}{6 M_{pl}^2} \times (0.3) T_R^3 \quad (\text{A11})$$

The yield from scattering is

$$Y_2 \simeq 2 \cdot 10^{-23} \left(\frac{T_R}{100 \text{MeV}} \right) A_2, \quad (\text{A12})$$

where

$$A_2 = \left(\frac{10}{g_*(T_R)} \right)^{3/2} \left(\ln \left(\frac{T_R^3}{m_A^2 m_e} \right) - 0.8 \right). \quad (\text{A13})$$

for each KK mode with mass m_A . This bound is valid only if A_2 is positive, i.e. the reheating temperature is significantly higher than the KK mode mass.

Class III: Thermal axion produced mainly by pion-pion scattering $\pi^\pm \pi^0 \rightarrow \pi^\pm a$, $\pi^+ \pi^- \rightarrow \pi^0 a$, for $T_R > 10 \text{ MeV}$.

$$\Gamma_{2\pi \rightarrow a\pi} Y_{EQ} = \frac{3}{1024 \pi^5} \frac{C_{a\pi}^2}{f_{PQ}^2 f_\pi^2} T^5 \left(\frac{45}{2 \pi^2 g_{*s}} \right) \times I(T) \quad (\text{A14})$$

where $C_{a\pi}$ is $(1-z)/3(1+z)$ with $z = m_u/m_d$. In the limit of $m_A = 0$, we can used the temperature dependent function $I(T)$ in Ref. [21]

$$I(T) \equiv \int dx_1 dx_2 \frac{x_1^2 x_2^2}{y_1 y_2} f(y_1) f(y_2) \int_{-1}^1 d\omega \frac{(s - m_\pi^2)^3 (5s - 2m_\pi^2)}{s^2 T^4} \quad (\text{A15})$$

where $f(y) = 1/(e^y - 1)$, $x_i = |\vec{p}_i|/T$, $y_i = E_i/T$ ($i = 1, 2$) and $s = 2(m_\pi^2 + T^2(y_1 y_2 - x_1 x_2 \omega))$. We justify ourselves using $m_A = 0$ limit by presenting the plot produced by computer (Fig. 5) which shows that the mass dependence of $I(T)$ is indeed small.

$I(T)$ is around 10^3 for $T > 50$ MeV, and it suppressed exponentially at $T < 10$ MeV. For $T \ll m_\pi$, we can approximate function $I(T)$,

$$I(T) = \frac{\pi}{8} \left(\frac{3m_\pi}{T} \right)^5 \exp \left(-2 \frac{m_\pi}{T} \right). \quad (\text{A16})$$

We can estimate thermal axion KK mode yield from pion scattering,

$$Y_3 \simeq 2 \cdot 10^{-3} \frac{M_{pl} T_R^3}{f_{PQ}^2 f_\pi^2} A_3 \simeq 6 \cdot 10^{-10} \left(\frac{T_R}{100 \text{ MeV}} \right)^3 \left(\frac{10^{12} \text{ GeV}}{f_{PQ}} \right)^2 A_3 \quad (\text{A17})$$

where

$$A_3 = \left(\frac{10}{g_*(T_R)} \right)^{3/2} C_{a\pi}^2 \left(\frac{I(T_R)}{1000} \right). \quad (\text{A18})$$

Class IV: Thermal axion can be generated through the two photon inverse decay,

$$\Gamma_{a_{KK} \rightarrow 2\gamma} \simeq \frac{C_{a\gamma}^2}{64\pi} \left(\frac{\alpha}{\pi} \right)^2 \frac{m_A^3}{f_{PQ}^2} \simeq 2.7 \cdot 10^{-8} C_{a\gamma}^2 \frac{m_A^3}{f_{PQ}^2}, \quad (\text{A19})$$

which leads

$$Y_4 \simeq 8 \cdot 10^{-10} \frac{M_{pl} T_R}{f_{PQ}^2} A_4 \simeq 2 \cdot 10^{-16} \left(\frac{10^{12} \text{ GeV}}{f_{PQ}} \right)^2 \left(\frac{T_R}{100 \text{ MeV}} \right) A_4 \quad (\text{A20})$$

where

$$A_4 = C_{a\gamma}^2 \left(\frac{10}{g_*(T_R)} \right)^{3/2} \left(\frac{m_A}{T_R} \right)^3. \quad (\text{A21})$$

This shows that the axion produced from the inverse decay will dominate over the axion from the pion scattering **III**, if $T_R < 10$ MeV. Thermal axion can be produce with photon electron scattering process **II**, we will ignore this axion unless the reheating temperature is very low (compared with the process **IV**, it should be less than a few MeV).

To estimate the energy density of the axion KK mode for given T , multiply the entropy density at the temperature $s(T)$ and the mass of axion KK mode to the yield. We should count the number of KK modes for the allowed energy range, $(Er_n)^N$ where E is typically T_R . But for case **III**, $T_R = \max\{m_\pi, T_R\}$. N is the dimension of the bulk where the produced particle exists. $N = n$ and $N = m$ for case $\{I, II\}$ and $\{III, IV\}$ respectively.

APPENDIX B: CALCULATION OF THE COSMOLOGICAL BOUNDS

In this section we describe the details of these calculations for two non-trivial cases, $n = 4, m = 3$ at $M_* = 1$ TeV and $n = 3, m = 2$ at $M_* = 10$ TeV, though we have calculated with computer all relevant cosmological bounds with allowed sets of m and n and presented it in Fig. 3 and 4.

This is the case with relatively low $T_R \leq \mathcal{O}(100)$ MeV, because of large $f_{PQ} \sim 10^{14}$ GeV. Since the life-time of the axion KK mode is quite long for $m_A < 100$ MeV and CMBR is stronger bound than the light element breaking bound, the bound from light element breaking is not relevant in these cases.

1. Big Bang Nucleosynthesis Bound

The axion KK mode with mass below 100 MeV cannot decay before 1 sec and will be restricted by BBN bound on neutrino species. Let's assume that $T_R > 10$ MeV then

$$\begin{aligned} \left. \frac{\rho_A}{s} \right|_{BBN} &\simeq D \int_{m_0}^{m_1} dm_A (m_A r_n)^m Y_A \\ &\simeq m_1 Y_3 (m_1 r_n)^m \sim 2 \cdot 10^{-3} \frac{M_{pl} T_R^3 m_1^{m+1}}{M_*^{m+2} f_\pi^2} A_3 < 0.1 \text{ MeV} \end{aligned} \quad (\text{B1})$$

where $m_1 = \max\{m_\pi, T_R\}$ and D is normalization constant defined as

$$D \int_{E_1}^{E_2} dm_A \times m_A^{m-1} r_n^m \equiv \# \text{ of KK modes between } E_1 \text{ and } E_2.$$

We used the torus compactification with uniform distance approximation $V_n = r_n^n$, $V_m = r_n^m$ so that $D = m$. This gives a bound for $n = 4, m = 3$ at 1 TeV

$$T_R < 90 A_3^{-1/3} \text{ MeV} \simeq 80 \text{ MeV} \quad (\text{B2})$$

and for $n = 3, m = 2$ at 10 TeV

$$T_R < 100 A_3^{-1/3} \text{ MeV} \simeq 90 \text{ MeV}. \quad (\text{B3})$$

2. Over-closure Bound

We can divide this bound by KK mass m_2 . If $m_A > m_2$, its life-time is shorter than the age of universe, otherwise it will remain as a cold dark matter. For $T_R < 10 \text{ MeV}$, the over-closure is not a problem in the region we are interested. Therefore Y_3 is the most dominant source of axion KK mode in this case. The total cold dark matter density will be

$$\frac{\rho_A}{s_0} \simeq D \int_{m_0}^{m_2} dm_A (m_A r_n)^m Y_3 < 3 \cdot 10^{-6} h^2 \text{ MeV} \quad (\text{B4})$$

where $m_2 \simeq 10 \text{ MeV}$, $m_0 = r_n^{-1} \simeq 30 \text{ keV}$ for $n = 4, m = 3$ at 1 TeV and $m_2 \simeq 1 \text{ MeV}$, $m_0 \simeq 5 \text{ keV}$ for $n = 3, m = 2$ at 10 TeV. Using Eq.(A17), Eq.(B4) becomes

$$2 \cdot 10^{-3} m_2^{m+1} \frac{M_{pl} T_R^3}{f_\pi^2 M_*^{m+2}} A_3 < 3 \cdot 10^{-6} h^2 \text{ MeV}. \quad (\text{B5})$$

We can get the bound on T_R for $n = 4, m = 3$ at 1 TeV,

$$T_R < 70 \left(\frac{h}{0.7} \right)^{2/3} A_3^{-1/3} \text{ MeV} \quad (\text{B6})$$

and for $n = 3, m = 2$ at 10 TeV

$$T_R < 350 \left(\frac{h}{0.7} \right)^{2/3} A_3^{-1/3} \text{ MeV}. \quad (\text{B7})$$

We can consider another situation that the axion KK mode which can decay into some relativistic dark matter X dominantly. In this case,

$$\frac{\rho_A}{s_0} \simeq D \int_{m_2}^{m_1} dm_A (m_A r_n)^m Y_3 \frac{T_0}{T(m_A)} < 3 \cdot 10^{-6} h^2 \text{ MeV}, \quad (\text{B8})$$

where $T_0 \sim 2 \times 10^{-13} \text{ GeV}$ is current temperature of universe and

$$T(m_A) \simeq \sqrt{\Gamma_{a_{KK} \rightarrow X} M_{pl}} \sim 2 \cdot 10^{-4} C_{a\gamma} Br^{-1/2} \frac{m_A^{3/2} M_{pl}^{1/2}}{f_{PQ}} \quad (\text{B9})$$

is the temperature when axion KK mode with mass m_A decays, where Br is the branching ratio for

$$Br = \frac{\Gamma_{a_{KK} \rightarrow 2\gamma}}{\Gamma_{a_{KK} \rightarrow X} + \Gamma_{a_{KK} \rightarrow 2\gamma}} \simeq \frac{\Gamma_{a_{KK} \rightarrow 2\gamma}}{\Gamma_{a_{KK} \rightarrow X}}, \quad (\text{B10})$$

which is the case that majority of axion KK modes decays into the invisible relativistic dark matter X . Then Eq.(B8) becomes

$$2 \cdot 10^{-9} Br^{1/2} \frac{f_{PQ}}{C_{a\gamma}} \frac{M_{pl}^{1/2} T_R^3 m_1^{m-1/2}}{M_*^{m+2} f_\pi^2} A_3 \text{ MeV} < 3 \cdot 10^{-6} h^2 \text{ MeV}. \quad (\text{B11})$$

For the reasonable range of Br , we can set $m_1 = m_\pi$. For $n = 4, m = 3$ at 1 TeV,

$$T_R < 12 Br^{-1/6} \left(\frac{h}{0.7} \right)^{2/3} C_{a\gamma}^{1/3} A_3^{-1/3} \text{ MeV} \simeq 30 \text{ MeV (for } Br = 1), \quad (\text{B12})$$

and for $n = 3, m = 2$ at 10 TeV,

$$T_R < 28 Br^{-1/6} \left(\frac{h}{0.7} \right)^{2/3} C_{a\gamma}^{1/3} A_3^{-1/3} \text{ MeV} \simeq 40 \text{ MeV (for } Br = 1). \quad (\text{B13})$$

3. Cosmological Microwave Background Radiation

CMBR bound is the most severe during the time period $10^6 < \tau_A < 10^{12}$ sec. The bound

$$\frac{\Delta\rho_\gamma}{s} \leq 2.5 \times 10^{-5} T_D \quad (\text{B14})$$

is actually weaker than other constraints if the branching ratio Br is large. In this case, the reheating temperature should be relatively small $T_R < 10$ MeV. But if Br is very small, it gives stronger bound than other cosmological constraints. T_D is the same as Eq.(B9). If we assume $T_R > 10$ MeV, and set $\Delta\rho_\gamma \simeq Br \times \rho_A(T_D)$, Eq.(B14) becomes

$$2 \cdot 10^{-3} Br \cdot m_A^{m+1} \frac{M_{pl} T_R^3}{M_*^{m+2} f_\pi^2} A_3 \leq 5 \cdot 10^{-9} C_{a\gamma} Br^{-1/2} \frac{M_{pl}^{1/2} m_A^{3/2}}{f_{PQ}} \quad (\text{B15})$$

If $T_R \leq m_\pi$, the maximal value of KK mode mass is around m_π . This leads the bound, for $n = 4, m = 3$ at 1 TeV,

$$T_R < 2 \cdot 10^{-2} Br^{-1/2} A_3^{-1/3} C_{a\gamma}^{1/3} \text{ MeV} \quad (\text{B16})$$

for $n = 3, m = 2$ at 10 TeV,

$$T_R < 4 \cdot 10^{-2} Br^{-1/2} A_3^{-1/3} C_{a\gamma}^{1/3} \text{ MeV}. \quad (\text{B17})$$

To have $T_R > 100 \text{ MeV}$, we need a very small branching ratio $\sim 10^{-7}$. (This bound is not valid if $T_R < 10 \text{ MeV}$.)

4. Diffused Photon Background

Since the life-time of axion KK mode is longer than 10^{14} sec in allowed region, we obtain most strong bound on reheating temperature from diffused photon background. Let's consider three cases,

Case 1. $T_R > 10 \text{ MeV}$: In this case, the majority of KK mode will decay before present time. The observed bound when $800 \text{ keV} < E < 30 \text{ MeV}$ is,

$$\frac{d\mathcal{F}}{d\Omega} < 78 \left(\frac{E}{1\text{keV}} \right)^{-1.4} \text{ cm}^{-2}\text{sr}^{-1}\text{sec}^{-1} \quad (\text{B18})$$

corresponds for theoretical prediction

$$\frac{d\mathcal{F}}{d\Omega} = \frac{n_\gamma c}{4\pi} \simeq Br \frac{Y_3 s_0 c}{4\pi} (m_A r_n)^m. \quad (\text{B19})$$

Here $s_0 c \simeq 9 \cdot 10^{13} \text{ cm}^{-2} \text{ sec}^{-1}$. This will give the inequality,

$$Br \times Y_3 (m_A r_n)^m < 6 \cdot 10^{-16} \left(\frac{E}{\text{MeV}} \right)^{-1.4} \quad (\text{B20})$$

where $E \simeq m_A / (2(1+z))$

$$1+z \simeq 4 \times 10^{11} \left(\Omega_0 h^2 \right)^{-1/3} \left(\frac{\tau_D}{\text{sec}} \right)^{-2/3}, \quad (\text{B21})$$

The time of axion KK mode decay into two photons is

$$\tau_D \simeq Br \cdot \Gamma_{a_{KK} \rightarrow 2\gamma}^{-1} \simeq 3.7 \cdot 10^7 Br \frac{f_{PQ}^2}{m_A^3}. \quad (\text{B22})$$

Thus the present energy of diffused photon (for $\Omega_0 \simeq 1$) is

$$\begin{aligned} E &\simeq 10^{-12} \left(\frac{\tau_D}{\text{sec}} \right)^{2/3} \left(\frac{h}{0.7} \right)^{2/3} m_A \\ &\simeq 10^{-18} C_{a\gamma}^{-4/3} Br^{2/3} \left(\frac{f_{PQ}}{\text{GeV}} \right)^{4/3} \left(\frac{h}{0.7} \right)^{2/3} \left(\frac{10\text{MeV}}{m_A} \right) \text{MeV}. \end{aligned} \quad (\text{B23})$$

Inserting this result into (B20) and setting $h = 0.7$ leads

$$2 \cdot 10^{-3} Br \frac{M_{pl} m_A^m T_R^3}{f_\pi^2 M_*^{m+2}} A_3 < 10^{10} \left(\frac{f_{PQ}}{\text{GeV}} \right)^{-1.9} C_{a\gamma}^{1.9} Br^{-0.9} \left(\frac{m_A}{10\text{MeV}} \right)^{1.4} \quad (\text{B24})$$

and it gives a bound for $n = 4, m = 3$ at 1 TeV,

$$T_R < 2 \cdot 10^{-2} \left(\frac{Br}{C_{a\gamma}} \right)^{-0.63} \left(\frac{m_A}{10\text{MeV}} \right)^{-0.53} A_3^{-1/3} \text{MeV} \quad (\text{B25})$$

and for $n = 3, m = 2$ at 10 TeV,

$$T_R < 3 \cdot 10^{-2} \left(\frac{Br}{C_{a\gamma}} \right)^{-0.63} \left(\frac{m_A}{10\text{MeV}} \right)^{-0.2} A_3^{-1/3} \text{MeV}. \quad (\text{B26})$$

Case 2. $T_R < 10 \text{ MeV}$, $Br < \Gamma_{a_{KK} \rightarrow 2\gamma}/H_0$:

In this case $Y \simeq Y_4$ and axion KK mode will decay before present time. $H_0 \simeq 2 \cdot 10^{-42} h$ GeV is Hubble constant of present universe. Then the relation with energy and life time become same as Eq.(B24),

$$\begin{aligned} Br \cdot Y_4 \cdot (m_A r_n)^m &= 8 \cdot 10^{-10} Br \frac{M_{pl} m_A^m T_R}{M_*^{m+2}} A_4 \\ &< 10^{10} \left(\frac{f_{PQ}}{\text{GeV}} \right)^{-1.9} C_{a\gamma}^{1.9} Br^{-0.9} \left(\frac{m_A}{10\text{MeV}} \right)^{1.4} \end{aligned} \quad (\text{B27})$$

and it gives a bound for $n = 4, m = 3$ at 1 TeV

$$T_R < 0.06 \left(\frac{Br}{C_{a\gamma}} \right)^{-1.9} \left(\frac{m_A}{\text{MeV}} \right)^{-1.6} A_4^{-1} \text{MeV} \quad (\text{B28})$$

and in $m_A \sim T_R$ limit,

$$T_R < 0.3 Br^{-0.73} \text{MeV}. \quad (\text{B29})$$

For $n = 3, m = 2$ at 10 TeV

$$T_R < 0.03 \left(\frac{Br}{C_{a\gamma}} \right)^{-1.9} \left(\frac{m_A}{\text{MeV}} \right)^{-0.6} A_4^{-1} \text{ MeV}, \quad (\text{B30})$$

and in $m_A \sim T_R$ limit,

$$T_R < 0.1 Br^{-1.2} \text{ MeV}. \quad (\text{B31})$$

Case 3. $T_R < 10 \text{ MeV}$, $Br > \Gamma_{a_{KK} \rightarrow 2\gamma}/H_0$:

In this case, axion KK modes will not decay before the present time. Then the theoretical prediction becomes (see references, for instance, Sec. 5.5 in [19])

$$\frac{d\mathcal{F}}{d\Omega} \simeq Br \times \frac{Y_4 s_0 c}{4\pi} \frac{\Gamma_{a_{KK} \rightarrow 2\gamma}}{Br H_0} \left(\frac{2E}{m_A} \right)^{3/2} (m_A r_n)^m \quad (\text{B32})$$

and this will lead

$$2 \cdot 10^{-17} C_{a\gamma}^2 \frac{M_{pl} T_R}{M_*^{m+2} H_0} \frac{m_A^{m+3/2} E^{3/2}}{f_{PQ}^2} A_4 < 6 \cdot 10^{-16} \left(\frac{E}{\text{MeV}} \right)^{-1.4}. \quad (\text{B33})$$

Approximately, for $n = 4, m = 3$ at 1 TeV,

$$T_R \left(\frac{m_A}{\text{MeV}} \right)^{9/2} < 2 \cdot 10^6 \left(\frac{h}{0.7} \right) C_{a\gamma}^{-2} A_4^{-1} \left(\frac{E}{\text{MeV}} \right)^{-2.9} \text{ MeV}. \quad (\text{B34})$$

We can approximate $m_A \sim T_R \sim E$, then $A_4 \sim C_{a\gamma}^2$ and

$$T_R < 5.5 \left(\frac{h}{0.7} \right)^{0.12} C_{a\gamma}^{-0.48} \text{ MeV}. \quad (\text{B35})$$

The case $n = 3, m = 2$ at 10 TeV, the majority of thermal axion KK mode with $T_R > 1 \text{ MeV}$ will decay before present time. The condition $Br \simeq \Gamma_{a_{KK} \rightarrow 2\gamma}/H_0$ and Eq.(B31) will determine the reheating temperature

$$T_R < 2 \sim 3 \text{ MeV}. \quad (\text{B36})$$

We did full calculations for all relevant sets of m and n with computer and get consistent results. For instance, you can see three different regions of DPBR bound in Fig. 1 and 2.

TABLES

TABLE I. PQ scales and lifetimes of axion and graviton KK modes for $M_* = 1$ TeV, where

$$M_{100} \equiv \frac{m_A}{100\text{MeV}}$$

(m, n)	$r_n^{-1}[\text{MeV}]$	$f_{PQ}[\text{GeV}]$	$\tau_g[\text{sec}]$	$\tau_a[\text{sec}]$
(1, 2)	4×10^{-10}	5×10^{10}	$2 \times 10^7 M_{100}^{-4}$	$6 \times 10^7 M_{100}^{-3}$
(2, 3)	6×10^{-5}	2×10^{13}	$1 \times 10^6 M_{100}^{-5}$	$8 \times 10^{12} M_{100}^{-3}$
(2, 4)	2×10^{-2}	5×10^{10}	$2 \times 10^{11} M_{100}^{-5}$	$6 \times 10^7 M_{100}^{-3}$
(3, 4)		3×10^{14}	$3 \times 10^7 M_{100}^{-6}$	$3 \times 10^{15} M_{100}^{-3}$
(2, 5)	0.7	1×10^9	$2 \times 10^{14} M_{100}^{-5}$	$5 \times 10^4 M_{100}^{-3}$
(3, 5)		2×10^{12}	$1 \times 10^{12} M_{100}^{-6}$	$7 \times 10^{10} M_{100}^{-3}$
(4, 5)		2×10^{15}	$9 \times 10^9 M_{100}^{-7}$	$1 \times 10^{17} M_{100}^{-3}$
(3, 6)	7	5×10^{10}	$2 \times 10^{15} M_{100}^{-6}$	$6 \times 10^7 M_{100}^{-3}$
(4, 6)		2×10^{13}	$1 \times 10^{14} M_{100}^{-7}$	$8 \times 10^{12} M_{100}^{-3}$

TABLE II. PQ scales and lifetimes of axion and graviton KK modes for $M_* = 10$ TeV.

(m, n)	$r_n^{-1}[\text{MeV}]$	$f_{PQ}[\text{GeV}]$	$\tau_g[\text{sec}]$	$\tau_a[\text{sec}]$
(1, 2)	4×10^{-8}	2×10^{11}	$2 \times 10^9 M_{100}^{-4}$	$6 \times 10^8 M_{100}^{-3}$
(2, 3)	3×10^{-3}	4×10^{13}	$3 \times 10^9 M_{100}^{-5}$	$4 \times 10^{13} M_{100}^{-3}$
(2, 4)	0.6	2×10^{11}	$2 \times 10^{14} M_{100}^{-5}$	$6 \times 10^8 M_{100}^{-3}$
(3, 4)		6×10^{14}	$1 \times 10^{12} M_{100}^{-6}$	$9 \times 10^{15} M_{100}^{-3}$
(2, 5)	20	6×10^9	$1 \times 10^{17} M_{100}^{-5}$	$8 \times 10^5 M_{100}^{-3}$
(3, 5)		4×10^{12}	$2 \times 10^{16} M_{100}^{-6}$	$5 \times 10^{11} M_{100}^{-3}$
(4, 5)		3×10^{15}	$4 \times 10^{15} M_{100}^{-7}$	$3 \times 10^{17} M_{100}^{-3}$
(3, 6)	160	2×10^{11}	$2 \times 10^{19} M_{100}^{-6}$	$6 \times 10^8 M_{100}^{-3}$
(4, 6)		4×10^{13}	$3 \times 10^{19} M_{100}^{-7}$	$4 \times 10^{13} M_{100}^{-3}$

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FIGURES

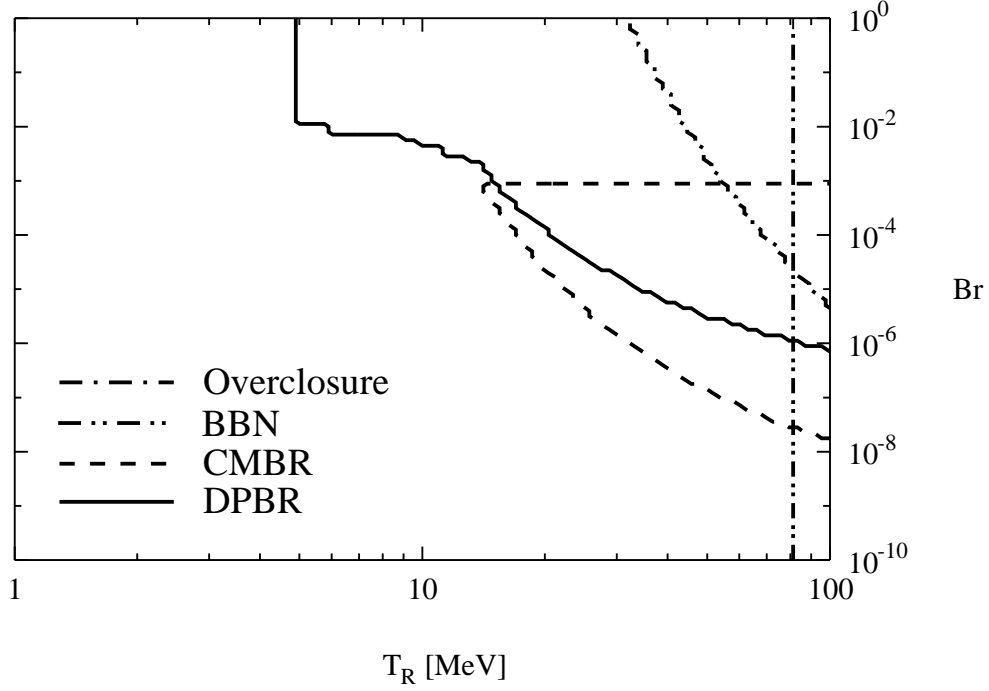


FIG. 1. The bound of T_R and Br for $M_* = 1$ TeV and $n = 4, m = 3$, The upper and right side of each line is excluded region.

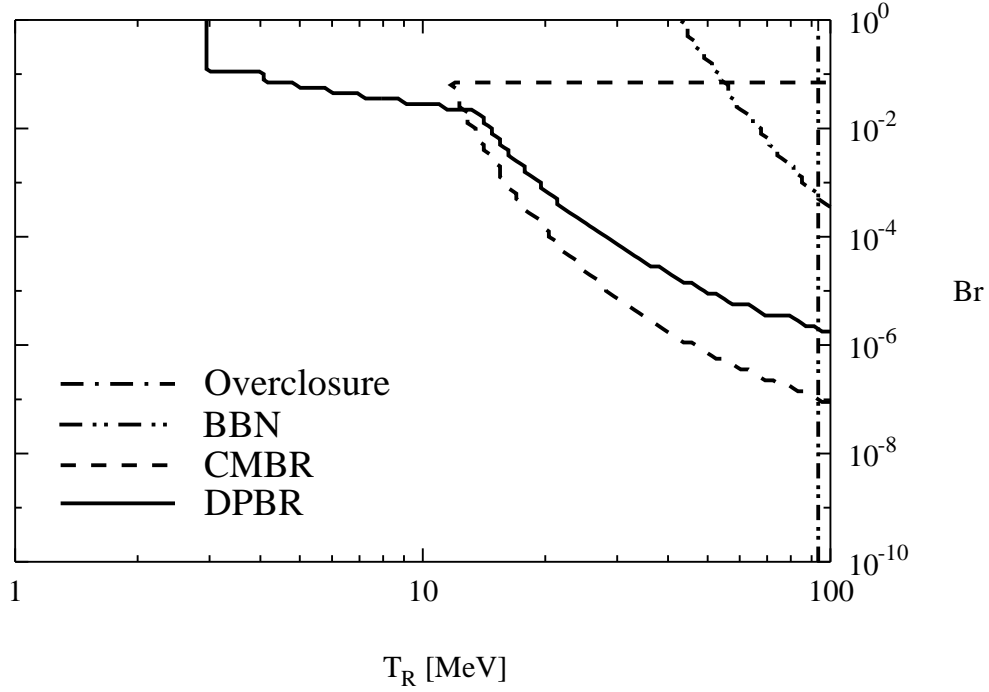


FIG. 2. The bound of T_R and Br for $M_* = 10$ TeV and $n = 3, m = 2$. The excluded region is same as Fig. 1.

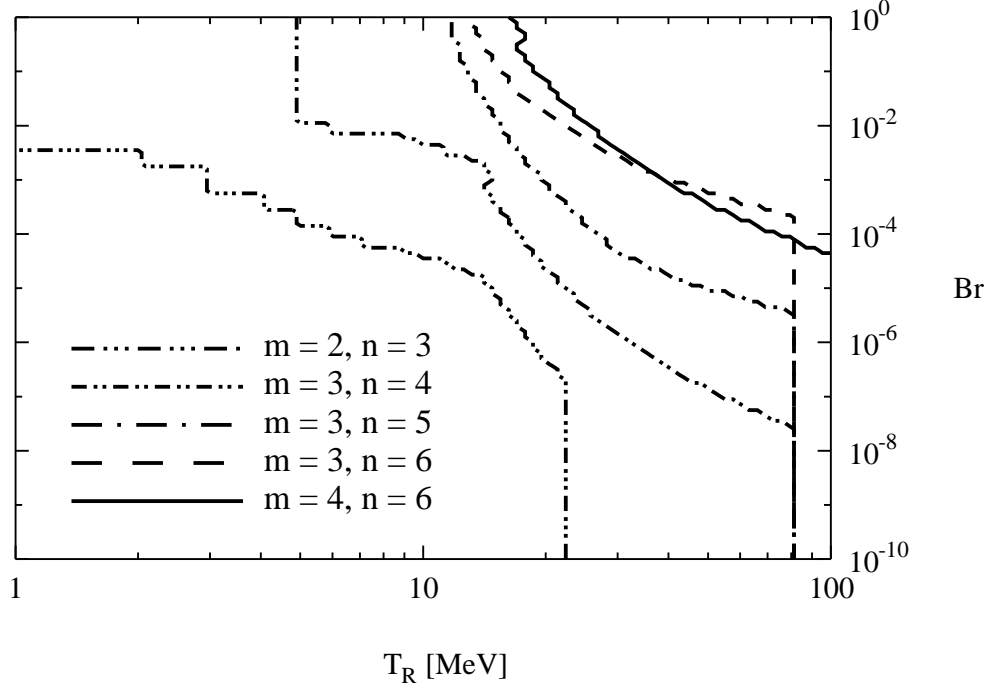


FIG. 3. The bound of T_R and Br for $M_* = 1$ TeV. The upper and right side of each line is excluded region.

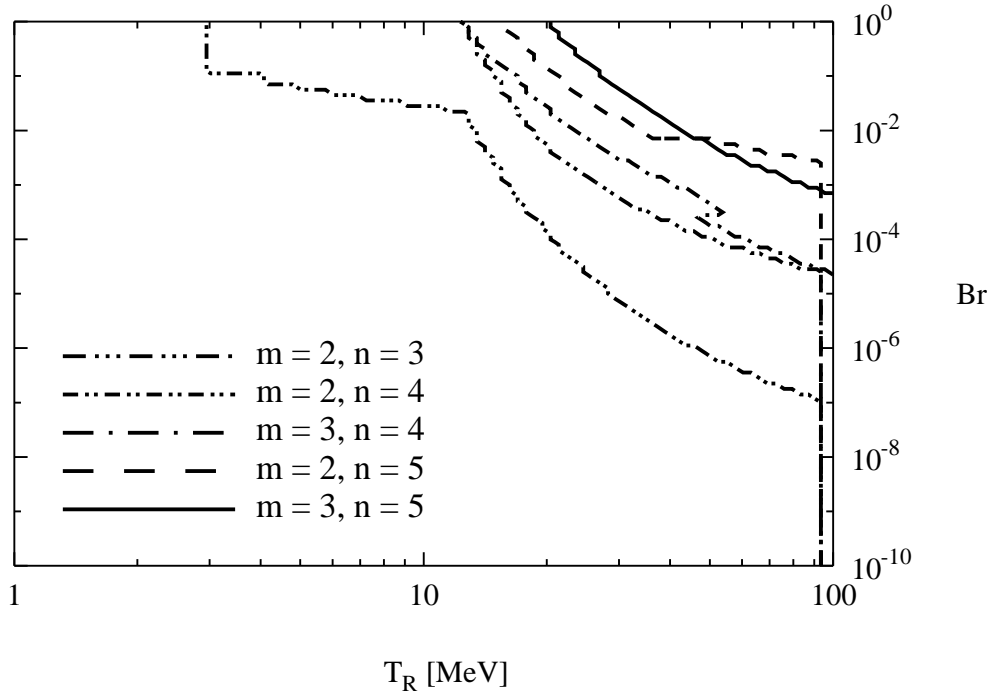


FIG. 4. The bound of T_R and Br for $M_* = 10$ TeV. The excluded region is same as Fig. 3.

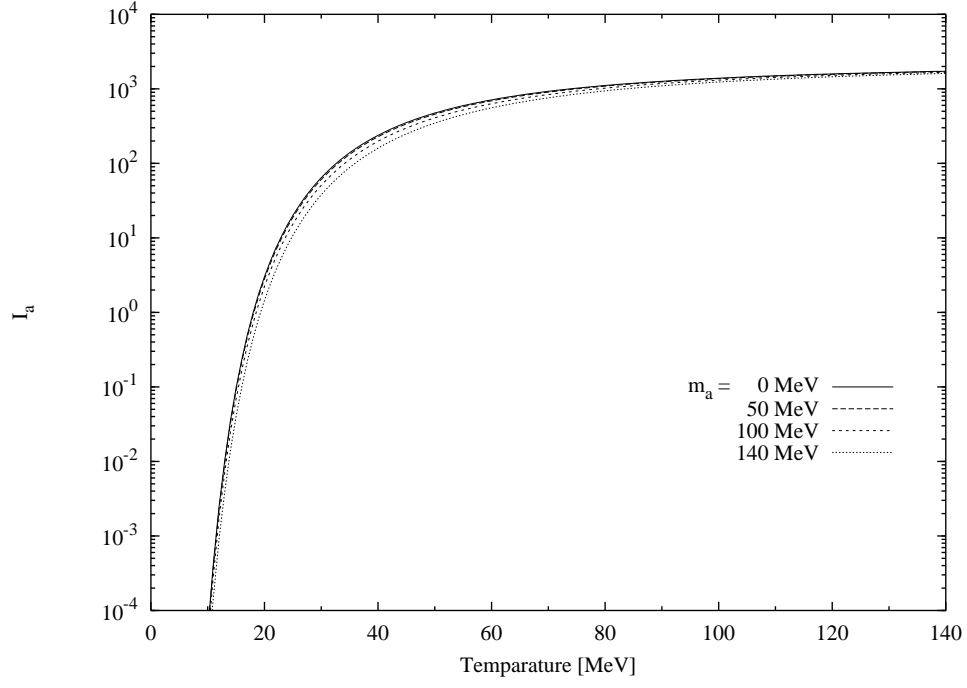


FIG. 5. The function $I(T)$ for various m_A in the KK mode yield III.